Linear combinations of vectors

Interpretation: Linear combination of vectors

Imagine that we have three 2D vectors v\_1, v\_2 and v\_3 in a plane:



Consider another vector, v\_4:



Is it possible to express v\_4 as a linear combination of vectors v\_1 and v\_2? Explain. (yes)

What about v\_1 and v\_3? (yes)

v\_2 and v\_3? (yes)

Is it possible to express v\_4 as a linear combination of all three vectors v\_1, v\_2 and v\_3? Explain. (yes, if it is possible with only two of them, it must be the case for all three too)

How would we write that? (vector equation)

Can we find some linear combination? If we can, let’s write some concrete linear combination. (yes, solving the vector equation through system of linear equations and perhaps using matrices will give infinitely many solutions because we have two linear equations with three variables)

What is the chance that we can write the 4th 2D vector as a linear combination of three 2D vectors? (big, in most cases there will be infinitely many possibilities)

Now imagine that we go one dimension up, to 3D. Imagine that we have four vectors, u\_1 = (1, 0, 1), u\_2 = (0, 2, 0), u\_3 = (-1, 1, 0) and u\_4 = (0, 1, 1).

Consider the 5th vector, u\_5 = (-2, 1, -1).

Is it possible to express u\_5 as a linear combination of u\_1, u\_2 and u\_3? Explain. (yes, they are linearly independent)

What about u\_2, u\_3 and u\_4? (yes, they are linearly independent)

Is it possible to express u\_5 as a linear combination of all four vectors u\_1, ..., u\_4? Explain. (yes, if it is possible with only three of them, it must be the case for all four too)

How would we write that? (vector equation)

Is it possible to express u\_5 as a linear combination of u\_1 and u\_2 only? (no)

What about u\_1 and u\_3? (yes)

u\_2 and u\_3? (no)

u\_2 and u\_4? (no)

u\_3 and u\_4? (yes)

Now think about vector v\_6 = (-2, 0, 2). Is it possible to express u\_6 as a linear combination of u\_1 and u\_2 only?

What about u\_1 and u\_3?

u\_2 and u\_3?

u\_2 and u\_4?

u\_3 and u\_4?

Now we try to guess: What is the chance that we can write a 3D vector w as a linear combination of two other 3D vectors u and v? (little, in most cases there will be no solution)

Do we see any geometrical hint what the vector w should satisfy to be a linear combination of vectors u and v? (it should lie in the linear span of u and v, in the linear subspace generated by those two vectors)

By linear combinations, we work with vectors. What about matrices? Do we see any links between linear combinations of vectors and matrices? (The linear combinations are represented by vector equations. It is possible to express them as systems of linear equations or by matrix equations.)

(How to get matrices there? The matrix equation Ax=b?

b is clear, that is the righthand side of the vector equation (the vector we want to "combine" from the others). How to motivate Ax?)

Actually, I think you have introduced matrices in the script/video, so I think we are done here.