# Theme 3: Linear combinations of vectors

Consider three vectors in the plane $\vec{v\_{1}}, \vec{v\_{2}}$ og $\vec{v\_{3}}$:



Now consider a fourth vector $\vec{v}\_{4}$:

1. Would it be possible to express $\vec{v}\_{4}$ as a linear combination of $\vec{v}\_{1}$ and $\vec{v}\_{2}$? What about $\vec{v}\_{1}$og $\vec{v}\_{3}$? And $\vec{v}\_{2}$ og $\vec{v}\_{3}$? (PS: it is not necessary to find the numbers to express these combinations)
2. Could we express $\vec{v}\_{4}$ as a linear combination of **all three** vectors $\vec{v}\_{1}, \vec{v}\_{2}$ og $\vec{v}\_{3}$? How could we write that? Try to find such a linear combination.
3. In general: What is the chance that we may express a fourth vector as a combination of the three others?

Suppose we now move to 3d. And suppose we have the vectors $\vec{u}\_{1}=[1,0,1]$, $\vec{u}\_{2}=[0,2,0]$, $\vec{u}\_{3}=[-1,1,0]$ and $\vec{u}\_{4}=[0,1,1]$. Also consider a vector $\vec{u}\_{5}=[-2,1,-1]$.

1. Is it possible to express $\vec{u}\_{5}$ as a linear combination of $\vec{u}\_{1}$, $\vec{u}\_{2}$ and $\vec{u}\_{3}$? What about $\vec{u}\_{2}$, $\vec{u}\_{3}$and $\vec{u}\_{4}$?
2. Is it possible to express $\vec{u}\_{5}$ as a linear combination of all the vectors $\vec{u}\_{1}$, $\vec{u}\_{2}$, $\vec{u}\_{3}$ and $\vec{u}\_{4}$? How would you express that?
3. Is it possible to express $\vec{u}\_{5} $as a linear combination of only two of the vectors?
	1. $\vec{u}\_{1}$ and $\vec{u}\_{2}?$
	2. What about $\vec{u}\_{1}$ and $\vec{u}\_{3}?$
	3. $\vec{u}\_{2}$ and $\vec{u}\_{3}$?
	4. $\vec{u}\_{2}$ and $\vec{u}\_{4}$?
	5. $\vec{u}\_{3}$ and $\vec{u}\_{4}$?
4. Now have a look at the vector $\vec{u}\_{6}=[-2,0,2]$? Is it possible to express this vector as a linear combination of
	1. $\vec{u}\_{1}$ and $\vec{u}\_{2}?$
	2. $\vec{u}\_{1}$ and $\vec{u}\_{3}?$
	3. $\vec{u}\_{2}$ and $\vec{u}\_{3}$?
	4. $\vec{u}\_{2}$ and $\vec{u}\_{4}$?
5. Now you can try to guess: What is the chance that we may express a 3d vector $\vec{w}$ as a linear combination of two other 3d vectors $\vec{u}$ og $\vec{v}$? Is there a “hint” that we may use from geometry that could come to our aid here?