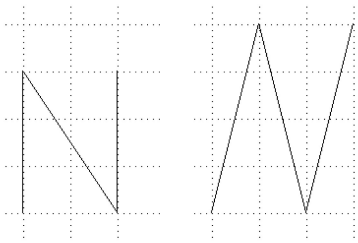
Skewed N (application of linear transformation)

pondělí 22. března 2021 16:32

u2t1

THE ITALICIZING N PROBLEM

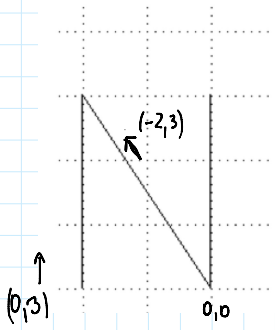
Name _____ Group Members _____

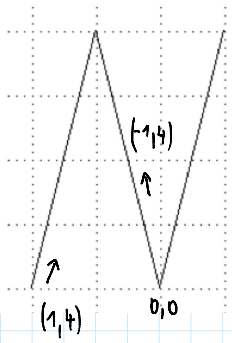


Suppose the "N" on the left is written in regular 12-point font. Find a matrix *A* that will transform N into the letter on the right, which is written in 'italics' in 16-point font.

A=

Work with a small group and write out your solution and approach. Make a list of any assumptions you notice your group making, or any questions for further pursuit that come to mind.





$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & -2 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 4 & 4 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 0 & -2 & 1 & 0 \\ 3 & 3 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 3 & 3 & 0 & 1 \\ 0 & -2 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 1 & 0 & 1/3 \\ 0 & 1 & -1/2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 1/2 & 1/3 \\ 0 & 1 & -1/2 & 0 \end{array}\right)$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 4 & 4 \end{pmatrix}\begin{pmatrix} 1/2 & 1/3 \\ -1/2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1/3 \\ 0 & 4/3 \end{pmatrix}$$

(need to make it more inquiry - raise questions)

Have you ever tried it with your students? If so, how did it work?

I was thinking about how would I start, how would I approach the task. I came to an idea that I need to find two vectors and calculate the matrix on them.
But then I have a problem because there are two fixed points in the mapping. In my opinion, this brings a need of one linear and one affine transformation (the matrix A would be the same for both in this case). Is that correct? If so, I think I would be able to solve it. I wonder how about first-year students.

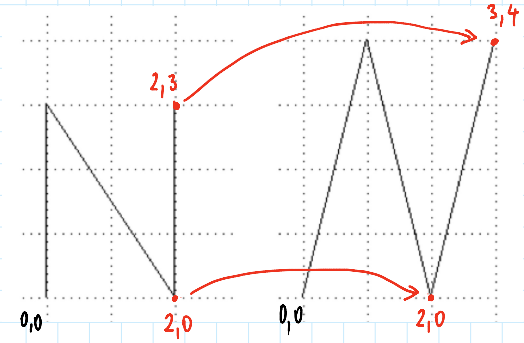
However, then I started thinking about the font size. I do not know how it works. I can imagine that if I make the change from 12 to 16, the height will increase by 4/3. What about the width? From what I see in the picture, I would say it increased by 3/2. I just wonder, does it work like that in reality?

OK, I see the students were smarter than me. :-)

But at least now I understand how it works.

It is not important where we choose the origin to be, if we calculate with relative vectors in the coordinate system. However, we need to keep the same orientation of those relative vectors in both pictures.

What about points?



$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 1 & 1/2 & 0 \\ \cdots & \cdots & \cdots & \cdots \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/3 \\ \cdots & \cdots & \cdots & \cdots \end{array}\right)$$

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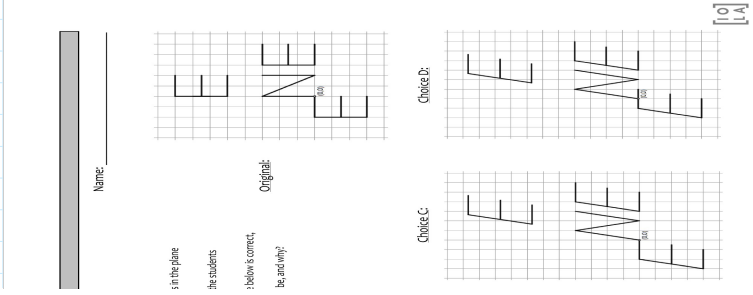
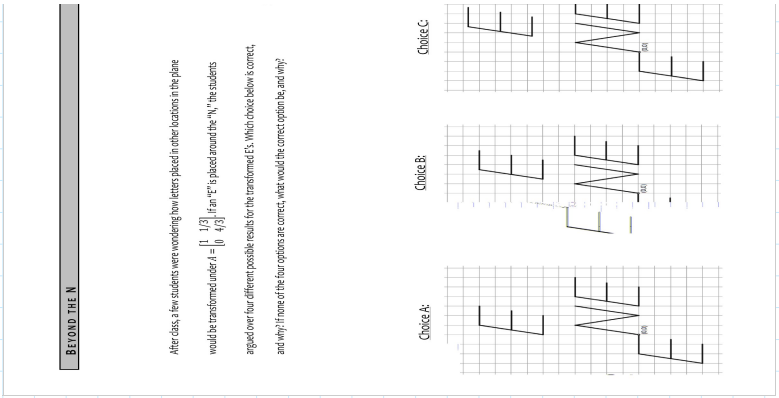


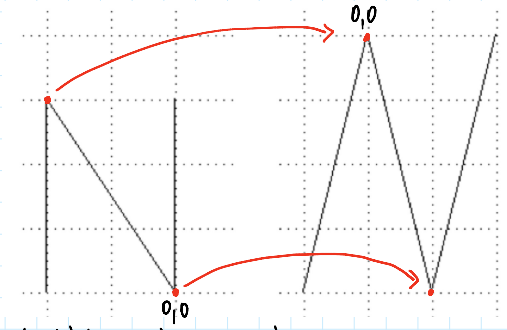


$$\left(\begin{array}{cc|cc} c & d & 0 & 3 \end{array}\right) = \left(\begin{array}{cc} 0 & 4 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 1 & 1/2 & 0 \\ 0 & 1 & 0 & 1/3 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/3 \\ 0 & 1 & 0 & 1/3 \end{array}\right)$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 1/2 & -1/3 \\ 0 & 1/3 \end{pmatrix} = \begin{pmatrix} 1 & 1/3 \\ 0 & 4/3 \end{pmatrix}$$

Points work too. What about different coordinates?



$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & -2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -4 & 0 \end{pmatrix}$$

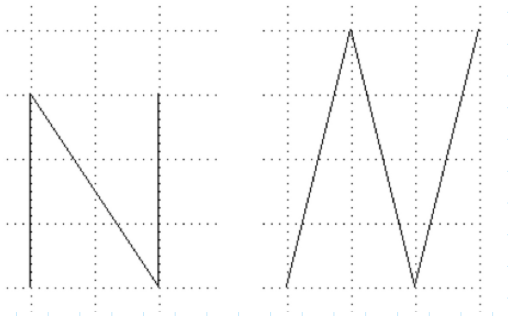
Does not work. Linear transformation must map 0 to 0.

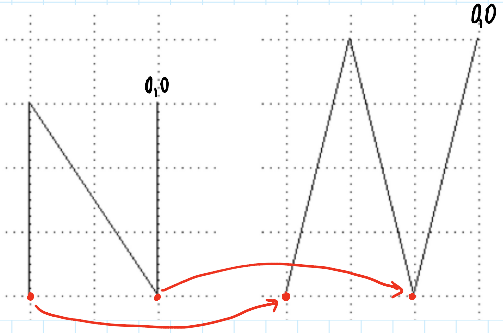
Is it possible to determine the matrix by looking at mapping of unit vectors? How could we do it practically?

How do we interpret the picture? As vectors represented by the lines, or as vectors represented by points?

Skewed/transformed "Q"

Task





$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & -2 \\ -3 & -3 \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ -4 & -4 \end{pmatrix}$$

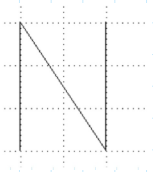
$$\left(\begin{array}{cc|cc} 0 & -2 & 1 & 0 \\ -3 & -3 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 1 & 0 & -1/3 \\ 0 & 1 & -1/2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/3 \\ 0 & 1 & -1/2 & 0 \end{array}\right)$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ -4 & -4 \end{pmatrix}\begin{pmatrix} 1/2 & -1/3 \\ -1/2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1/3 \\ 0 & 4/3 \end{pmatrix}$$

Different coordinates are fine as long as 0 is mapped to 0.

Suppose that the "N" on the left is written in regular 12-point font, and the "N" on the right is written in "italics" in 16-point font.

Are we able to find the (linear) transformation that transforms the "N" on the left into the "N" on the right?
How could we do that?
How would the transformation look like?/How could we write the transformation mathematically?/Which mathematical objects/concepts could we use to write down the transformation?

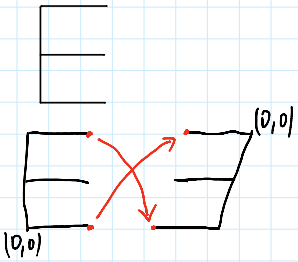


How would this letter be transformed under

$$A = \begin{pmatrix} -3/2 & 0 \\ 0 & 5/3 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & -1/3 \\ 0 & -1 \end{pmatrix}$$

$$C = \begin{pmatrix} -1/2 & 1 \\ -1 & 0 \end{pmatrix}$$



$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -2 & -3 \\ 0 & -3 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 1 & 1/2 & 0 \\ 0 & 1 & 0 & 1/3 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/3 \\ 0 & 1 & 0 & 1/3 \end{array}\right)$$

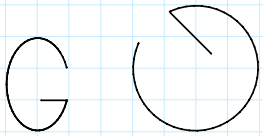
$$A = \begin{pmatrix} -2 & -3 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 1/2 & -1/3 \\ 0 & 1/3 \end{pmatrix} = \begin{pmatrix} -1 & -1/3 \\ 0 & -1 \end{pmatrix}$$

$C = \begin{pmatrix} & \\ -1 & 0 \end{pmatrix}$

?

What do the transformations A, B and C to the letter "Q" below?

Are we able to find the (linear) transformation that transforms the "G" on the left into the "G" on the right?



$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ -2 & 0 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 1/3 \end{array}\right)$$

$$A = \begin{pmatrix} -1 & 3 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 0 & 1/3 \end{pmatrix} = \begin{pmatrix} -1/2 & 1 \\ -1 & 0 \end{pmatrix}$$