$$\left| \tfrac{}{4} \qquad -\tfrac{1}{20} - \lambda \right|$$

$$= \lambda^2 + \left(\frac{3}{4} + \frac{1}{20}\right)\lambda + \cancel{\frac{3}{80}} - \cancel{\frac{3}{80}} = \lambda\left(\lambda + \frac{16}{20}\right) = \lambda(\lambda + 0.8)$$

$$\lambda_1 = 0$$

$$\lambda_2 = \frac{-8}{10} = -0.8$$

$$\lambda_1 = 0: \begin{pmatrix} -\frac{3}{4} & \frac{1}{20} \\ \frac{3}{4} & -\frac{1}{20} \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = 0 \qquad \left(\begin{array}{cc|c} -\frac{3}{4} & \frac{1}{20} & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad \begin{array}{l} y_1 = t \\ x_1 = \frac{1}{15} t \end{array}$$

FOR EXAMPLE $t = 15$: $x_1 = 1,\ y_1 = 15$

$$(1, 15)^T$$

$$\lambda = \frac{-8}{10}: \left(\begin{array}{cc|c} -\frac{3}{4} + \frac{4}{5} & \frac{1}{20} & 0 \\ +\frac{3}{4} & -\frac{1}{20} + \frac{4}{5} & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} \frac{1}{20} & \frac{1}{20} & 0 \\ \frac{3 \cdot 5}{4 \cdot 5} & \frac{15}{20} & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} \frac{1}{20} & \frac{1}{20} & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\begin{array}{l} y_2 = s \\ x_2 = -s \end{array} \qquad s = -1: \ x_2 = 1,\ y_2 = -1 \qquad (1, -1)^T$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = X = c_1 \begin{pmatrix} 1 \\ 15 \end{pmatrix} e^{0t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-0.8t} = c_1 \begin{pmatrix} 1 \\ 15 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-0.8t}$$

$$A = c_1 + c_2 e^{-0.8t}$$

$$B = 15c_1 - c_2 e^{-0.8t}$$

$$A(0) = 1 \qquad\qquad 1 = c_1 + c_2 \rightarrow c_2 = 1 - c_1$$

$$B(0) = 0.2 = \frac{1}{5} \qquad\qquad \frac{1}{5} = 15c_1 - c_2$$

$$\frac{1}{5} = 15c_1 - (1 - c_1)$$

$$\frac{1}{5} + 1 = 16c_1$$

$$\frac{6}{5} = 16c_1$$

$$c_1 = \frac{6}{5 \cdot 16} = \frac{3}{40}$$

$$c_2 = 1 - \frac{3}{40} = \frac{37}{40}$$

$$A = \frac{3}{40} + \frac{37}{40} e^{-0.8t}$$

$$B = \frac{45}{40} - \frac{37}{40} e^{-0.8t}$$

Hmm, this will amount to stable values of A=3/40=0.075 and B=45/40=1.125, which is the stable solutions found using the Markov model?

Yes. The constants fit well.

BUT WHAT EXACTLY IS t ?

The (continuous) solution is not correct. It does not reflect reality. The problem is that the MODEL is wrong. I mean, the system

$$\frac{dA}{dt} = -0.75\,A + 0.05\,B$$

$$\frac{dB}{dt} = 0.75\,A - 0.05\,B$$

does not describe the situation given by the task.

This system says that at any instant, A decreases about 0.75 ~~units~~ of A and increases about 0.05 ~~units~~ of B. Similarly for B: it increases about 0.75 ~~units~~ of A and decreases about 0.05 ~~units~~ of B at any instant.
That does not correspond to

lutions[2] are two real-life examples. In every minute, 75 mole % of A are converted to B, and 5% of B converted to A. How many moles of A and B are

And why do this not correspond? I fail to see the logical breach? It doesn't say that it decrease 0.75 **units** of A, it says 0.75 **all** A. If you say this is wrong, how would you say a correct model using differentials would look like, if one would attempt this?

You are right, "units" was a bad expression. It should indeed be "whole" A.
However, it does not change anything on the fact that the model is wrong and does not correspond to the problem.
The problem says that the changes happen "in every minute". The model says that the changes happen "at any instant".

The correct (discrete) model of the reaction is

The Markov chain model of the reaction is

$$A(n+1) = 0.25\,A(n) + 0.05\,B(n) \qquad (1a)$$

$$B(n+1) = 0.75\,A(n) + 0.95\,B(n) \qquad (1b)$$

How should the continuous model look like?

$$X_{n+1} = \underbrace{\begin{pmatrix} \frac{1}{4} & \frac{1}{20} \\ \frac{3}{4} & \frac{19}{20} \end{pmatrix}}_{S} X_n$$

This is another formulation of the Markov model, OK.

Yes. Just using fractions, not decimals. More convenient for "hands on" calculations.

$$\det(S-\lambda I) = \begin{vmatrix} \frac{1}{4}-\lambda & \frac{1}{20} \\ \frac{3}{4} & \frac{19}{20}-\lambda \end{vmatrix} =$$

$$= \lambda^2 - \left(\frac{1}{4}+\frac{19}{20}\right)\lambda + \frac{19}{80} - \frac{3}{80} =$$

$$= \lambda^2 - \frac{24}{20}\lambda + \frac{16}{80}$$

$$= \lambda^2 - \frac{6}{5}\lambda + \frac{1}{5}$$

$$= (\lambda-1)\left(\lambda-\frac{1}{5}\right)$$

$$\lambda_1 = 1$$

$$\lambda_2 = \frac{1}{5}$$

$$\lambda_1 = 1: \begin{pmatrix} \frac{1}{4}-1 & \frac{1}{20} \\ \frac{3}{4} & \frac{19}{20}-1 \end{pmatrix} \sim \begin{pmatrix} -\frac{3}{4} & \frac{1}{20} \\ 0 & 0 \end{pmatrix} \quad \begin{matrix} y = t \\ x = \frac{1}{15}t \end{matrix} \qquad -\frac{3}{4}x = -\frac{1}{20}t$$

$$t = 15: \; v_1 = (1, 15)^T$$

$$\lambda_2 = \frac{1}{5}: \begin{pmatrix} \frac{1}{4}-\frac{1}{5} & \frac{1}{20} \\ \frac{3}{4} & \frac{19}{20}-\frac{1}{5} \end{pmatrix} = \begin{pmatrix} \frac{1}{20} & \frac{1}{20} \\ \frac{3}{4} & \frac{3}{4} \end{pmatrix} \quad \begin{matrix} y = s \\ x = -s \end{matrix}$$

$$s = -1: \; v_2 = (1, -1)^T$$

$$\begin{pmatrix} A(n) \\ B(n) \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 15 \end{pmatrix} 1^n + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \left(\frac{1}{5}\right)^n$$

This looks like a reasonable extension to a solution of discrete difference equations for two unknowns, but I haven't worked with such before. Do you have a reference?

I learned this during my study. One of the very few things that I remember. :-)
Basic reference:
https://en.wikipedia.org/wiki/Linear_difference_equation#Solution_with_distinct_characteristic_roots

$$A(0) = 1 \qquad 1 = c_1 + c_2$$

$$B(0) = \frac{1}{5} \qquad \frac{1}{5} = 15c_1 - c_2$$

$$\frac{6}{5} = 16c_1$$

$$c_1 = \frac{3}{40}$$

$$c_2 = 1 - \frac{3}{40} = \frac{37}{40}$$