Mars exploration (application of linear combination of vectors)

pondělí 22. března 2021 16:31

I would carefully distinguish between „any point may be reached“ and „linear independence“. As it is now, it might lead to a false conclusion that vectors are linearly independent if any point may be reached. I suggest that we say something like „as we can see in the Geogebra, linear span of the three vectors is three-dimensional (3<=>3), and that means linear independence“. We may also add "Otherwise, which means that the number of vectors is higher than the number of dimensions of their span, we call them linearly dependent“. (The option of number of vectors less than the number of dimensions of their span is not possible.)

The last part of the script definitely needed a bit closer attention. I agree that it was misleading as it was stated. However, the terms linear independence and Span hasn’t been defined in Session 1, reason being that it takes a bit time to make the formal definition appear. Thus, I was thinking to introduce them in a bit informal manner in this video, but it is not easy without stating a bit dubious sentences 😉 I think I have corrected this now. I might also discuss “linear dependence” as you say, but it require even more space on theory.

Would it be possible for you to extend the theory in your exercise? For example, show how a plane in R^3 may be spanned by only two R^3 vectors, which should make great sense for the students. This should fit well with the task you wanted to extend this video with, that is, one of the thrusters being damaged and what that leads to in practice.

Additionally, four R^3 vectors would lead to the fact that one of them would be linearly dependent of the others. This may be also built into the exercise by considering the possibility of adding two extra thrusters to fix the problem. Just my ideas 😉

It seems we have one topic - Mars exploration - for both the video and the task. I see we have a couple of concepts around that topic: linear combination, linear independence, linear dependence, linear span. I think that linear combination has been covered in Session 1 to some extent. So, we should think about how we distribute the remaining three concepts between the video and the task. My feeling is that we should choose one concept for the video and leave two concepts for the task. And as the video will come to the students first, the concepts in the task might be dependent on/derived from the concept of the video but not vice versa. As I see it now, this gives a possibility to mention „linear span“ in the video („any point may be reached means that the span of the three thrusters is 3-dimensional“) and let the students explore linear dependence/independence in the task.

I will limit my video script on the mars landing to evolve around linear span, and then you may organize a task that goes further towards linear dependence/independence. I think that would be a great mapping from the video domain into the task co-domain 😊

Task

Imagine that we have a vessel with only two thrusters working, and the other two were damaged. The two thrusters that work can move the vessel in the directions v1=[1,1,-1] and v2=[1,-1,1].

(1, 1, -1)
(1, -1, 1)

We have a few missions to accomplish:

1. We are starting from [0,0,0] and we shall land at [3,0,0]. Is it possible with the only two thrusters we have? (Are we able to land at [3,0,0]?) If so, how could the coefficients/scale factors be interpreted in terms of space/time? If not, how would we fix the shuttle/how would we fix the thruster(s)/how many thrusters and in which directions we need to add?

2. We are starting from [0,0,0] and we shall land at [-3,0,0]. Is it possible with the only two thrusters we have? (Are we able to land at [-3,0,0]?) If so, how could the coefficients/scale factors be interpreted in terms of space/time? If not, how would we fix the shuttle/how would we fix the thruster(s)/how many thrusters and in which directions we need to add?

3. We are starting from [0,0,0] and we shall land at [-3,-3,-3]. Is it possible with the only two thrusters we have? (Are we able to land at [-3,-3,-3]?) If so, how could the coefficients/scale factors be interpreted in terms of space/time? If not, how would we fix the shuttle/how would we fix the thruster(s)/how many thrusters and in which directions we need to add?

Now imagine that we have a vessel with four thrusters that can move the vessel in the following directions:

u_1 = (1, 0, 1), u_2 = (0, 2, 0), u_3 = (-1, 1, 0) and u_4 = (0, 1, 1)

Where in the whole 3D space could we get with those thrusters? (Suppose we can move "frem og tilbake" with each thruster.)
What happens if one of the thrusters gets damaged? Where could we get? Does it make any difference which of the thrusters is damaged?

Where could we get = what is the span.

It does make difference because 1,3 and 4 are linearly dependent.

How do we anticipate students' discovery of this? I guess the best way is to explore this In geogebra, if they don't "see it" directly by just looking at the numbers in the coordinates?

Yes, I think it is a convenient approach, especially for students familiar with Geogebra. I expect that the use of Geogebra will speed up their exploration.

Finally, imagine that we have a vessel with four thrusters that can move the vessel in the following directions:

v_1 = (1, 0, -1), v_2 = (1, 0, 1), v_3 = (0, 1, 0) and v_4 = (0, -1, 1)

Where in the whole 3D space could we get with those thrusters? (Suppose we can move "frem og tilbake" with each thruster.)
What happens if one of the thrusters gets damaged? Where could we get? Does it make any difference which of the thrusters is damaged?

Where could we get = what is the span.

It does not make difference because all 3-element subsets consisting of 3 vectors are linearly independent = span the whole 3D space.

(1, 0, -1)
(1, 0, 1)
(0, 1, 0)
(0, -1, 1)

How do we anticipate students' discovery of this? Same line of thought as above?

Yes. :-)

If we compare the two cases with four thrusters, what can we say about the "range/span" of the vessel and "substitutability/non-substitutability" of particular thrusters?
Is there any way how to make use of vectors, matrices, lines, planes etc.?